

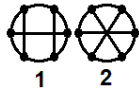
# Circular chord diagrams without loops

**2 chords**



1

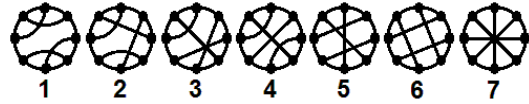
**3 chords**



1

2

**4 chords**



1

2

3

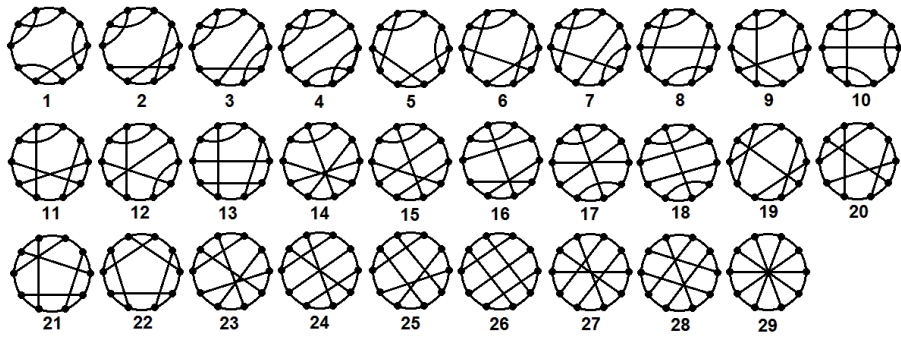
4

5

6

7

**5 chords**



1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

21

22

23

24

25

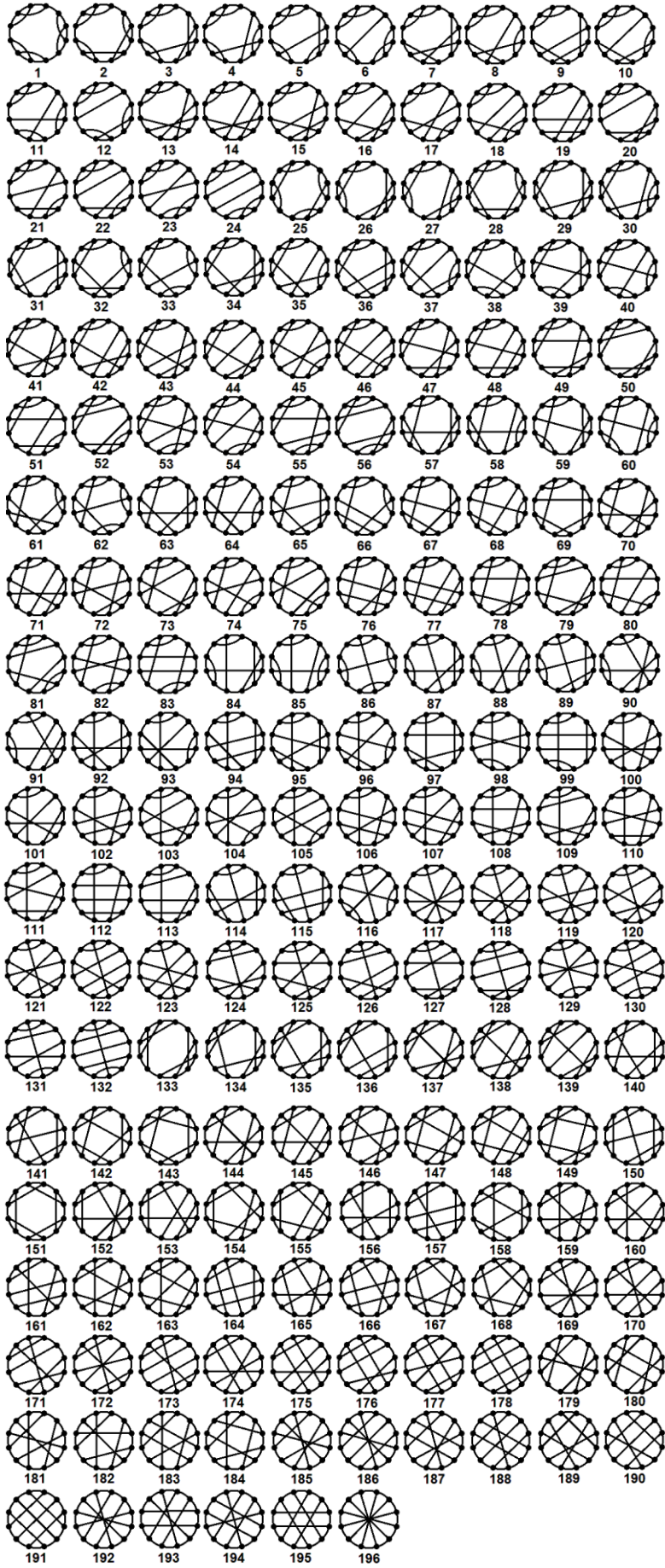
26

27

28

29

6 chords



# Relations

## 2 chords

$$[1]$$

## 3 chords

$$[1]$$

$$[2] = 2[1]$$

## 4 chords

$$[1]$$

$$[2]$$

$$[3] = 2[2] - [1]$$

$$[4] = [2] - [1]$$

$$[5] = [6] + [2] - [1]$$

$$[6]$$

$$[7] = [6] + 3[2] - 2[1]$$

## 5 chords

$$[1] = [3]$$

$$[2] = 2[3]$$

$$[3]$$

$$[4] = 0$$

$$[5] = [8]$$

$$[6] = 2[8] - [3]$$

$$[7] = [8] - [3]$$

$$[8]$$

$$[9] = 2[8] - 2[3]$$

$$[10] = [8] - [3]$$

$$[11]$$

$$[12] = 2[8] - 2[3]$$

$$[13] = [11] - [8] + [3]$$

$$[14] = [11] + 2[8] - 3[3]$$

$$[15] = [11] - [3]$$

$$[16] = 2[8] - 3[3]$$

$$[17] = 2[8] - 3[3]$$

$$[18] = [8] - 2[3]$$

$$[19] = 4[8] - 4[3]$$

[20]

$$[21] = [20] - [11] + 2[8] - [3]$$

$$[22] = [20] - 2[11] + 4[8] - 2[3]$$

$$[23] = 2[20] - [11]$$

$$[24] = 2[20] - 2[11]$$

$$[25] = 2[20] - [11] - 2[8] + 3[3]$$

$$[26] = 2[20] - 2[11] - [8] + 2[3]$$

$$[27] = 4[20] - 3[11] - 2[8] + 3[3]$$

$$[28] = 4[20] - 4[11] - 2[8] + 4[3]$$

$$[29] = 4[20] - 2[11]$$

## 6 chords

$$[1] = -[3] + 2[2]$$

[2]

[3]

$$[4] = [3] - [2]$$

$$[5] = [2]$$

$$[6] = -[3] + 2[2]$$

[7]

$$[8] = [3]$$

$$[9] = [7] - [3] + [2]$$

$$[10] = [2]$$

$$[11] = [2]$$

$$[12] = 0$$

$$[13] = [7] + [3]$$

$$[14] = [7]$$

$$[15] = [7]$$

$$[16] = [3]$$

$$[17] = [3]$$

$$[18] = [3] - [2]$$

$$[19] = [7] - [3] + [2]$$

$$[20] = 0$$

$$[21] = [2]$$

$$[[22]] = 0$$

$$[[23]] = -[[3]] + 2[[2]]$$

$$[[24]] = 0$$

[[25]]

$$[[26]] = 2[[25]]$$

$$[[27]] = [[25]]$$

[[28]]

$$[[29]] = 2[[28]] - [[2]]$$

$$[[30]] = [[28]] - [[2]]$$

$$[[31]] = [[28]]$$

$$[[32]] = 2[[28]] - [[25]] - [[2]]$$

$$[[33]] = [[28]] - [[25]]$$

[[34]]

$$[[35]] = 2[[28]] - [[25]] - [[2]]$$

$$[[36]] = [[34]] - [[28]] + [[2]]$$

$$[[37]] = [[28]] - [[25]]$$

$$[[38]] = [[28]] - [[2]]$$

$$[[39]] = 2[[28]] - [[25]] - [[2]]$$

$$[[40]] = [[28]] - [[25]]$$

$$[[41]] = [[34]] + 2[[28]] - 2[[25]] - [[2]] =$$

$$[[42]] = [[34]] - [[25]]$$

$$[[43]] = [[34]] - [[25]]$$

$$[[44]] = 2[[28]] - 2[[25]] - [[2]]$$

$$[[45]] = 2[[28]] - 2[[25]] - [[2]]$$

$$[[46]] = [[28]] - [[25]] - [[2]]$$

$$[[47]] = [[34]]$$

$$[[48]] = [[34]] - [[28]] + [[2]]$$

$$[[49]] = 2[[28]] - [[2]]$$

$$[[50]] = 2[[25]]$$

$$[[51]] = [[28]]$$

$$[[52]] = [[25]]$$

$$[[53]] = 2[[28]] - [[25]] - [[2]]$$

$$[[54]] = [[28]] - [[25]]$$

$$[[55]] = [[28]] - [[2]]$$

$$[[56]] = [[25]]$$

$$[[57]] = 4[[28]] - 2[[25]] - [[3]] - 2[[2]]$$

$$[[58]] = 2[[28]] - [[25]] - [[3]] - [[2]]$$

$$[[59]] = 2[[28]] - [[3]] - [[2]]$$

$$[[60]] = [[28]] - [[3]]$$

$$[[61]] = [[34]] - [[7]] + [[3]]$$

$$[[62]] = 2[[28]] - 2[[25]] - [[2]]$$

[[63]]

$$[[64]] = [[34]] - [[25]] - [[7]] + [[3]]$$

$$[[65]] = [[63]] - [[34]] + 2[[28]] - [[25]] + [[7]] - [[3]] - [[2]]$$

$$[[66]] = [[34]] - [[28]] + [[25]] - [[7]] + [[3]]$$

$$[[67]] = [[63]] - [[34]] + 2[[28]] - [[2]]$$

$$[[68]] = [[34]] - [[28]] - [[7]] + [[3]] + [[2]]$$

$$[[69]] = [[63]] - 2[[34]] + 4[[28]] - [[25]] + [[7]] - [[3]] - 2[[2]] *$$

$$[[70]] = 2[[63]] - [[34]] + [[3]]$$

$$[[71]] = 2[[63]] - 2[[34]] + [[7]]$$

$$[[72]] = [[63]]$$

$$[[73]] = 4[[28]] - 2[[25]] - [[3]] - 2[[2]]$$

$$[[74]] = [[34]] - [[25]] - [[7]] + [[3]]$$

$$[[75]] = 2[[28]] - [[25]] - [[3]] - [[2]]$$

$$[[76]] = 2[[63]] - [[34]] - 2[[28]] + 2[[25]] + [[3]] + [[2]]$$

$$[[77]] = 2[[63]] - 2[[34]] - [[28]] + [[25]] + [[7]] + [[2]]$$

$$[[78]] = [[63]] - [[34]] + 2[[28]] - [[2]]$$

$$[[79]] = 2[[28]] - [[3]] - [[2]]$$

$$[[80]] = [[34]] - [[28]] - [[7]] + [[3]] + [[2]]$$

$$[[81]] = [[28]] - [[3]]$$

$$[[82]] = [[34]] - [[7]] + [[3]]$$

$$[[83]] = [[34]] - [[28]] + [[25]] - [[7]] + [[3]]$$

$$[[84]] = 2[[28]] - 2[[25]] - [[2]]$$

$$[[85]] = [[28]] - [[25]] - [[2]]$$

$$[[86]] = [[28]] - [[3]]$$

$$[[87]] = [[34]] - [[25]] - [[7]] + [[3]]$$

$$[[88]] = 2[[28]] - [[25]] - [[3]] - [[2]]$$

$$[[89]] = [[34]] - [[28]] - [[7]] + [[3]] + [[2]]$$

$$\begin{aligned}
[[90]] &= [[34]] + 2[[28]] - 2[[25]] - [[7]] - [[2]] \\
[[91]] &= [[34]] - [[7]] \\
[[92]] &= 2[[63]] - [[34]] \\
[[93]] &= [[34]] + 2[[28]] - 2[[25]] - [[7]] - [[2]] \\
[[94]] &= 2[[63]] - [[34]] - 2[[28]] + [[25]] + [[3]] + [[2]] \\
[[95]] &= [[63]] - [[7]] + [[3]] \\
[[96]] &= [[34]] - [[25]] - [[7]] + [[3]] \\
[[97]] &= [[63]] - [[34]] + 2[[28]] - [[25]] - [[2]] \\
[[98]] &= [[34]] - [[7]] \\
[[99]] &= [[34]] - [[28]] - [[7]] + [[3]] \\
[[100]] &= 4[[63]] - 3[[34]] - 2[[28]] + 2[[25]] + 2[[3]] + [[2]] \\
[[101]] &= 2[[63]] - [[34]] \\
[[102]] &= 4[[63]] - 4[[34]] - 2[[28]] + 3[[25]] + [[7]] + [[3]] + [[2]] \\
[[103]] &= [[63]] - [[7]] + [[3]] \\
[[104]] &= [[34]] + 2[[28]] - 2[[25]] - [[7]] - [[2]] \\
[[105]] &= [[34]] - [[7]] \\
[[106]] &= 4[[63]] - 4[[34]] - 2[[28]] + 3[[25]] + [[7]] + [[3]] + [[2]] \\
[[107]] &= 2[[63]] - [[34]] - 2[[28]] + [[25]] + [[3]] + [[2]] \\
[[108]] &= 2[[63]] - [[34]] - 2[[28]] + 2[[25]] - [[7]] + 2[[3]] + [[2]] \\
[[109]] &= [[34]] - [[7]] \\
[[110]] &= 2[[63]] - [[34]] - 2[[28]] + 2[[25]] - [[7]] + 2[[3]] + [[2]] \\
[[111]] &= [[63]] - [[34]] + 2[[28]] - [[25]] - [[2]] \\
[[112]] &= 2[[63]] - 2[[34]] - [[28]] + 2[[25]] + [[3]] \\
[[113]] &= [[34]] - [[28]] - [[7]] + [[3]] \\
[[114]] &= 2[[63]] - 2[[34]] + [[7]] \\
[[115]] &= 2[[63]] - 2[[34]] - [[28]] + [[25]] + [[7]] + [[2]] \\
[[116]] &= 2[[28]] - 2[[25]] - [[2]] \\
[[117]] &= 4[[63]] - 2[[34]] - [[7]] + [[3]] \\
[[118]] &= 4[[63]] - 3[[34]] - 2[[28]] + 2[[25]] + [[3]] + [[2]] \\
[[119]] &= 4[[63]] - 3[[34]] - 2[[28]] + 2[[25]] + [[3]] + [[2]] \\
[[120]] &= 2[[63]] - [[34]] - [[7]] + [[3]] \\
[[121]] &= 2[[63]] - [[34]] - [[7]] + [[3]] \\
[[122]] &= 2[[63]] - 2[[34]] + [[25]] \\
[[123]] &= 4[[63]] - 4[[34]] - 2[[28]] + 2[[25]] + [[7]] + [[3]] + [[2]]
\end{aligned}$$

$$[[124]] = [[34]] + 2[[28]] - 2[[25]] - [[7]] - [[3]] - [[2]]$$

$$[[125]] = [[63]] - [[25]] - [[7]] + [[3]]$$

$$[[126]] = [[34]] - [[25]] - [[7]]$$

$$[[127]] = 4[[28]] - 4[[25]] - [[3]] - 2[[2]]$$

$$[[128]] = 2[[28]] - 2[[25]] - [[3]] - [[2]]$$

$$[[129]] = [[34]] + 2[[28]] - 2[[25]] - [[7]] - [[3]] - [[2]]$$

$$[[130]] = [[34]] - [[25]] - [[7]]$$

$$[[131]] = 2[[28]] - 2[[25]] - [[3]] - [[2]]$$

$$[[132]] = [[28]] - [[25]] - [[3]]$$

$$[[133]] = 4[[25]]$$

$$[[134]] = 4[[28]] - [[3]] - 2[[2]]$$

$$[[135]] = 2[[34]] - [[7]]$$

$$[[136]] = 2[[34]] - 2[[28]] - [[7]] + [[3]] + [[2]]$$

$$[[137]] = 2[[34]] + 4[[28]] - 4[[25]] - [[7]] - [[3]] - 2[[2]]$$

$$[[138]] = 2[[34]] - 2[[25]] - [[7]]$$

$$[[139]] = 4[[28]] - 4[[25]] - [[3]] - 2[[2]]$$

[[140]]

$$[[141]] = [[140]] - [[63]] + [[34]] - [[25]]$$

[[142]]

$$[[143]] = [[142]] - [[63]] + [[34]]$$

$$[[144]] = [[142]] + [[140]] - 4[[28]] + [[25]] + [[3]] + 2[[2]]$$

$$[[145]] = [[142]] + [[140]] - [[63]] - 4[[28]] + [[25]] + [[3]] + 2[[2]]$$

$$[[146]] = [[140]] - [[25]]$$

$$[[147]] = [[142]] + [[140]] - 2[[34]] + [[25]] + [[7]]$$

$$[[148]] = [[142]] + [[140]] - [[63]] - [[34]] - 2[[28]] + [[25]] + [[7]] + [[2]]$$

$$[[149]] = [[142]] + [[25]]$$

$$[[150]] = [[140]] - 2[[63]] + 3[[34]] - 3[[28]] - [[7]] + [[3]] + 2[[2]]$$

$$[[151]] = [[142]] - 2[[63]] + 3[[34]] - 3[[28]] + [[25]] - [[7]] + [[3]] + 2[[2]]$$

$$[[152]] = [[142]] + [[140]] - 2[[63]] + 2[[34]] - 2[[28]] - [[25]] + [[2]]$$

$$[[153]] = [[142]] + [[140]] - 3[[63]] + 3[[34]] - 4[[28]] - [[7]] + [[3]] + 2[[2]]$$

$$[[154]] = [[142]] + [[140]] - 2[[63]] + 2[[28]] - [[25]] + [[7]] - [[3]] - [[2]]$$

$$[[155]] = [[142]] + [[140]] - 3[[63]] + 2[[34]] - 2[[28]] + [[2]]$$

$$[[156]] = [[142]] + 3[[140]] - 6[[63]] + 5[[34]] - 8[[28]] + [[25]] - [[7]] + [[3]] + 4[[2]]$$

$$[[157]] = [[142]] + 3[[140]] - 6[[63]] + 4[[34]] - 8[[28]] + [[25]] + [[3]] + 4[[2]]$$



$$\begin{aligned}
[[158]] &= [[142]] + [[63]] - 4[[28]] + 2[[25]] - [[7]] + 2[[3]] + 2[[2]] \\
[[159]] &= 2[[142]] + 4[[140]] - 6[[63]] + [[34]] - 8[[28]] + 2[[25]] + [[7]] + [[3]] + 4[[2]] \\
[[160]] &= [[142]] + 2[[140]] - 2[[63]] - 4[[28]] + [[25]] + [[3]] + 2[[2]] \\
[[161]] &= 2[[142]] + 4[[140]] - 7[[63]] + [[34]] - 8[[28]] + 2[[25]] + 2[[7]] + 4[[2]] \\
[[162]] &= 2[[142]] + 3[[140]] - 5[[63]] + [[34]] - 8[[28]] + 3[[25]] + [[7]] + [[3]] + 4[[2]] \\
[[163]] &= [[142]] + 2[[140]] - 3[[63]] + [[34]] - 4[[28]] + [[25]] + [[3]] + 2[[2]] \\
[[164]] &= [[142]] + 3[[140]] - 6[[63]] + 3[[34]] - 7[[28]] + [[25]] + [[7]] + 4[[2]] \\
[[165]] &= 2[[142]] + 4[[140]] - 8[[63]] + 3[[34]] - 6[[28]] + [[7]] + 3[[2]] \\
[[166]] &= 2[[142]] + 4[[140]] - 9[[63]] + 4[[34]] - 8[[28]] + [[25]] + [[7]] + 4[[2]] \\
[[167]] &= 2[[142]] + 3[[140]] - 8[[63]] + 5[[34]] - 6[[28]] + 3[[2]] \\
[[168]] &= [[142]] + 2[[140]] - 4[[63]] + [[34]] - [[25]] + [[7]] - [[3]] \\
[[169]] &= 3[[142]] + 5[[140]] - 8[[63]] + 2[[34]] - 12[[28]] + 3[[25]] + [[7]] + [[3]] + 6[[2]] \\
[[170]] &= 3[[142]] + 5[[140]] - 10[[63]] + 3[[34]] - 12[[28]] + 3[[25]] + [[7]] + [[3]] + 6[[2]] \\
[[171]] &= 2[[142]] + 2[[140]] - 2[[63]] - 3[[34]] - 4[[28]] + 2[[25]] + 2[[7]] + 2[[2]] \\
[[172]] &= 2[[142]] + 2[[140]] - 2[[63]] - 3[[34]] - 4[[28]] + 2[[25]] + 2[[7]] + 2[[2]] \\
[[173]] &= 2[[142]] + 2[[140]] - 4[[63]] - [[34]] - 4[[28]] + 2[[25]] + [[7]] + 2[[2]] \\
[[174]] &= 3[[142]] + 5[[140]] - 8[[63]] + [[34]] - 14[[28]] + 5[[25]] + 2[[7]] + 2[[3]] + 7[[2]] \\
[[175]] &= 3[[142]] + 5[[140]] - 10[[63]] + 2[[34]] - 12[[28]] + 4[[25]] + 2[[7]] + [[3]] + 6[[2]] \\
[[176]] &= 2[[142]] + 2[[140]] - 2[[63]] - 3[[34]] - 6[[28]] + 4[[25]] + 2[[7]] + [[3]] + 3[[2]] \\
[[177]] &= 2[[142]] + 2[[140]] - 2[[63]] - 3[[34]] - 6[[28]] + 4[[25]] + 2[[7]] + [[3]] + 3[[2]] \\
[[178]] &= 2[[142]] + 2[[140]] - 4[[63]] - [[34]] - 5[[28]] + 3[[25]] + [[7]] + [[3]] + 2[[2]] \\
[[179]] &= 2[[142]] + 2[[140]] - 4[[34]] - 4[[28]] + 2[[25]] + 2[[7]] + [[3]] + 2[[2]] \\
[[180]] &= 2[[142]] + 2[[140]] - 4[[34]] - 8[[28]] + 6[[25]] + 2[[7]] + 2[[3]] + 4[[2]] \\
[[181]] &= 5[[142]] + 9[[140]] - 20[[63]] + 7[[34]] - 18[[28]] + 3[[25]] + 2[[7]] + [[3]] + 9[[2]] \\
[[182]] &= 4[[142]] + 7[[140]] - 16[[63]] + 6[[34]] - 14[[28]] + 2[[25]] + 2[[7]] + 7[[2]] \\
[[183]] &= 5[[142]] + 9[[140]] - 21[[63]] + 7[[34]] - 18[[28]] + 4[[25]] + 3[[7]] + 9[[2]] \\
[[184]] &= 3[[142]] + 6[[140]] - 14[[63]] + 6[[34]] - 10[[28]] + [[25]] + [[7]] + 5[[2]] \\
[[185]] &= 7[[142]] + 13[[140]] - 30[[63]] + 11[[34]] - 24[[28]] + 3[[25]] + 3[[7]] + 12[[2]] \\
[[186]] &= 7[[142]] + 13[[140]] - 34[[63]] + 14[[34]] - 22[[28]] + [[25]] + 3[[7]] - 2[[3]] + 11[[2]] \\
[[187]] &= 7[[142]] + 13[[140]] - 32[[63]] + 12[[34]] - 24[[28]] + 3[[25]] + 4[[7]] - [[3]] + 12[[2]] \\
[[188]] &= 7[[142]] + 13[[140]] - 36[[63]] + 16[[34]] - 22[[28]] + 3[[7]] - 2[[3]] + 11[[2]] \\
[[189]] &= 5[[142]] + 11[[140]] - 28[[63]] + 14[[34]] - 20[[28]] + [[25]] + [[7]] + 10[[2]] \\
[[190]] &= 5[[142]] + 11[[140]] - 30[[63]] + 15[[34]] - 18[[28]] - [[25]] + 2[[7]] - 2[[3]] + 9[[2]] \\
[[191]] &= 5[[142]] + 11[[140]] - 32[[63]] + 17[[34]] - 17[[28]] - 3[[25]] + 2[[7]] - 3[[3]] + 9[[2]]
\end{aligned}$$

$$[[192]] = 10[[142]] + 18[[140]] - 42[[63]] + 15[[34]] - 36[[28]] + 6[[25]] + 5[[7]] + 18[[2]]$$

$$[[193]] = 10[[142]] + 18[[140]] - 46[[63]] + 18[[34]] - 34[[28]] + 4[[25]] + 5[[7]] - [[3]] + 17[[2]]$$

$$[[194]] = 10[[142]] + 18[[140]] - 46[[63]] + 18[[34]] - 34[[28]] + 4[[25]] + 5[[7]] - [[3]] + 17[[2]]$$

$$[[195]] = 10[[142]] + 18[[140]] - 50[[63]] + 22[[34]] - 32[[28]] + 2[[25]] + 4[[7]] - 2[[3]] + 16[[2]]$$

$$[[196]] = 10[[142]] + 18[[140]] - 38[[63]] + 13[[34]] - 36[[28]] + 6[[25]] + 4[[7]] + [[3]] + 18[[2]]$$

